Real-world Problem Solving in Small Groups: Interaction Patterns of Third and Fourth Graders

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The study reported in this paper is concerned with the investigation of primary children's real-world problem solving where particular research interest relates to the dynamics of group problem solving in an authentic classroom setting. Initial interpretative analyses of the video-taped group interactions revealed a kind of "interweaving" of the individuals' lines of thought. The application of an interaction model from social psychology lead to the identification of four basic types of interaction sequences in the group work episodes.

The 4-year study which provides the scientific background of this paper was initiated because of the concern that despite a vast amount of research literature on (real-world) problem solving in the last two decades, still little is known about primary pupils' real-world problem strategies as related to:

- their original mathematical modeling processes, and
- the dynamics of group problem solving and peer interaction during group work in an authentic classroom setting.

This paper focuses on the second major strand of the study—the analysis of group work episodes from authentic third and fourth grade classrooms with the aim of identifying interaction patterns which help to elucidate the dynamics of social mathematical learning processes in problem solving. Although we are reasonably aware of the individual capabilities of single students, the complexity of the 'normal' classroom in terms of the mixed abilities, the impact of the variety of socio-cultural experiences of children, and their quantitative and qualitative participation in classroom interaction have been almost completely neglected.

Learning in Small Groups: Interaction, Communication, Cooperation

Current research literature on learning mathematics suggests that the investigation of this phenomenon should not be limited to the analysis of internal mental processes of knowledge acquisition of individuals. Learning is also a social process that frequently occurs in the interaction of individuals (see e.g., Cobb & Bauersfeld, 1995; Bruner, 1996; Steffe, Nesher, Cobb, Goldin & Greer, 1996; Boaler, 2000). Interactive processes are integral elements of learning; that is, learning is socially constituted (Krummheuer, 1997). Interdisciplinary approaches in this context frequently include the concept of communication, since interaction is dependent on communication. An individual who forms a relationship with another person and in doing so conveys information. Communication is however, also dependent on interaction. A person who informs another person about something at the same time influences this other person. Therefore, the content of interpersonal behavior can be described as communication, while interaction is concerned with the observable actions of individuals (Crott, 1979). Graumann (1972)

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distinguishes these two concepts in relation to respective research approaches. Thus, communication theories aim at the method of information transfer, while the scientific interest of interactionist theories is concerned with why and how individuals interact with one another.

Curriculum documents and teacher handbooks that take into consideration the implications of interaction and communication with peers for individual as well as social learning (see also Australian Education Council, 1991; Johnson & Johnson, 1991, Davidson, 1990) emphasise the importance of group work:

> Education research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: "examine", "represent", "transform", "solve", "apply", "prove", "communicate". This happens most readily when students work in groups, engage in discussions, make presentations, and in other ways take charge of their own learning. (National Research Council, 1989, p. 58-59)

In approaches to group work in the mathematics classroom however, peer interaction is frequently associated with cooperative behavior and collaborative skills of the group members. In that, cooperation is often associated with a certain kind of interaction and expectations regarding the respective behavior of the group members, as for example described by Johnson and Johnson (1991), who defined cooperation as: "Working together to accomplish shared goals and maximise one's own and others achievement. Individuals perceiving that they can reach their goals if the other group members also do" (p. 230).

While there seems to be a shared understanding in the scientific community that interaction with peers is an important factor for mathematics learning, the assumption that getting children to work in small groups automatically results in collaborative behaviour is obviously a myth. In her analyses of task-related verbal interactions with respect to mathematics learning in small groups, Webb (1991) found differences in terms of the level of mathematics learning in different group settings:

> The optimum small group setting is one in which students freely admit what they do and do not understand, consistently give each other detailed explanations about how to solve the problems, and give each other opportunities to demonstrate their level of understanding (p. 386).

Furthermore, Yackel, Cobb and Wood (1993) have pointed out the importance of social norms in the classroom, claiming that "the interactions that occur when children work together in small groups are intimately related to the children's mathematical conceptions as well as to the social norms that have been negotiated in the classroom".

Only very few studies on group work have attempted to study in detail the interactions that are taking place (Davidson & Lambdin Kroll, 1991). In the "Handbook of Research on Mathematics Teaching and Learning" published in 1992, Good, Mulryan and McCaslin stated a lack of understanding as to how students acquire problem solving skills with respect to group work. One of the studies, that informed the research reported in this paper, is the work of Kieran and Dreyfus (1998) on collaborative problem solving. With respect to the mathematical discourse of two high-ability 13-year-old boys, they identified five types of interaction in the analysis of the conditions under which the partner directed talk was or was not beneficial.

However, comprehensive scientific evidence in terms of *which* interactive processes underlie the different types of learning that take place during group work is to date yet to be provided. In this context, the classes of social interaction in terms of contingency developed by the social psychologists Jones and Gerard (1967) can provide a helpful starting point for respective research studies in the field of mathematics education.

A Social-psychological View of Interaction in Terms of Contingency

The approach chosen by Jones and Gerard in the late sixties attempted to classify conversation with respect to the way in which the social exchange of ideas—the interaction—occurs and in how far the behavior of one person is affected by that of another person during this process. Basis of their analysis is the dyad—the smallest possible interaction unit:

> Once a conversation begins, social contingency is immediately present. To some extent, thereafter, the next response of person A will be contingent on the last response of person B. B's next response will be in turn be determined by the preceding A response, and so it goes. The plans of each actor continue, however, to exert some influence on the unfolding responses, so that person A in effect responds to his own last response as well as to the last response of B. The response is thus jointly determined by self-produced and socially produced stimulation ... this way of looking at social versus internal determinants is helpful in classifying types of interaction (p. 506).

Continuing their concern with the degree to which social interactions involve true social influence, Jones and Gerard identify four classes of interactions that vary in the patterns of weights to be attached to self-produced versus social stimulation: pseudo, asymmetrical reactive, and mutual contingency. In the following schematic representations of these four classes persons A and B are partners in the interaction that flows through a series of responses. According to Jones and Gerard, "the solid arrows reflect the predominant source of influence or response determination, the dotted arrows reflect the less important or minor source" (p. 506).

The first class of interaction—*pseudocontingency*—can be described as a limited case of social interaction. The existence of a contingency between A and B appears, but in fact social stimuli are only minimally involved, since both actors predominantly follow their own pre-established plans.

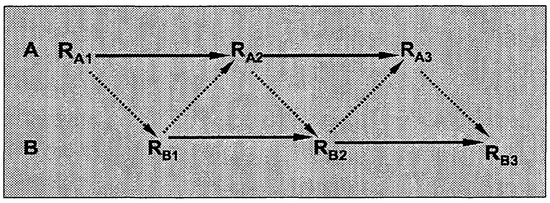

Figure 1. First class of social interaction: Pseudocontingency.

In the second class—*asymmetrical contingency*—the responses of person A are largely determined by self-produced stimuli (plans, strategies), whereas B's responses are determined predominantly by social stimuli produced by person A.

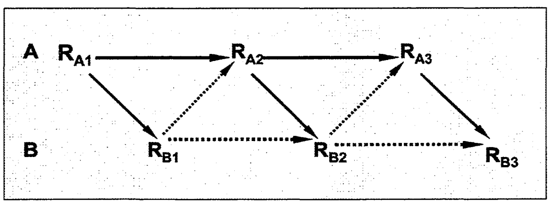

Figure 2. Second class of social interaction: Asymmetrical contingency.

According to Jones & Gerard, "it is at least logically possible that interaction could occur in a sequence in which each actor's response is almost entirely contingent on the preceding response of the other" (p. 510). In this situation neither A nor B follow self-produced stimuli, their responses are rather characterised by a *reactive contingency* and therefore closely related to and dependent on preceding social stimuli.

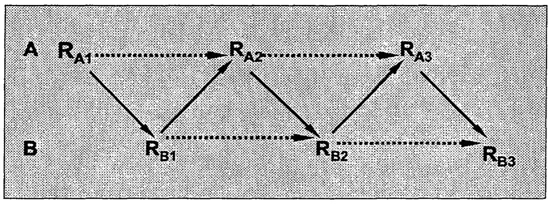

Figure 3. Third class of social interaction: Reactive contingency.

Only the fourth class of interaction is mutually driven. Each response is partially determined by the preceding response of the other and partly by the individual's internal and self-produced stimulation. Interactions on this *mutual contingency* class "thus require that a plan governs the responses of each actor, but the plan becomes continually recast in the light of the other's responses" (p. 511).

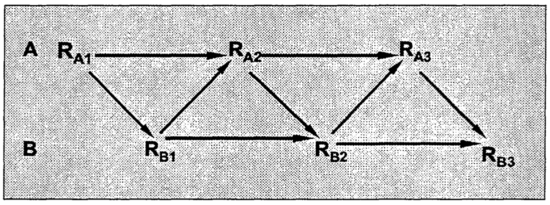

Figure 4. Fourth class of social interaction: Mutual contingency.

The classification of social interaction in terms of contingency as suggested by Jones and Gerard allows a more detailed study of phenomena that are frequently summarised under the heading 'social interaction', because it brings to the forefront the interweaving roles of self-stimulation and social stimulation in dyadic interaction. In the following samples it has been used to show how the application of this dyadic classification can be applied to larger social units—in this case groups of 3 to 5 pupils—to enhance our understanding of the types of interaction that occur during group work in the primary mathematics classroom.

Methodology and Data Analysis

The methodological framework of the research project underlying this paper is based on the interpretative classroom research approach (e.g., Jungwirth, Steinbring, Voigt, & Wollring, 2001) and involves pre-service teachers as teacher researchers. Consequently, the data collection and interpretation phases of the study followed a strict procedure consisting of four stages: video recordings, comprehensive transcriptions with respect to either the full document or selected segments of the recording relevant to a particular research question, the *sequential interpretation* of the data by a team of four student teachers under the guidance of the student teacher in charge of this data sample, and the *specific interpretation* of the results on the basis of relevant literature and research findings by the respective 'student teacher researcher' (for details regarding this procedure see Peter-Koop & Wollring, 2001). Each of the 23 student teachers involved in the study was responsible for the analysis of a specific sub-question in relation to a data sample (i.e., in most cases that data sample consisted of the work of one group of children), while the presentation, connection and discussion of the findings of these sub-studies in a type of meta-analysis was the responsibility of the project supervisor and author of this paper.

For the data collection, one grade 3 and two grade 4 mathematics classes from three different urban schools in a large city in north-western Germany were split into small groups, these groups were simultaneously videotaped while solving the task. Each group was videotaped on four occasions while solving a particular open real-world problem. Two student teachers assisted each group. In turns they either adopted the role of the teacher, assisting the group work if necessary, or they controlled the video recording. Once all groups had finished their work, the children presented, discussed, and evaluated the solution process and the respective results of each group in form of a "strategies conference" which was also videotaped.

The grouping of the children was organised in two different ways. In the first two classrooms (one grade 3 and one grade 4) the mathematics teacher was asked to form mixed-ability groups, because literature suggests that this is an optimal setting for group work. Furthermore, these groups were mostly heterogeneous in terms of gender. In the third classroom (grade 4) however, the teacher suggested that the children form their own groups. She argued that from her experience, the children would be in a position to know best with whom of their classmates they can work well. Thus, in this classroom groups were formed by the children primarily on the basis of friendship. Three out of four groups were gender homogenous. In addition, the members of two of the homogenous groups (one male, one female) were all low achievers.

Context of the Study: Open Real-world Problems

The real-world problems used in the study should intrinsically present challenges and thus motivate peer interaction during the solution process as opposed to problems that can be solved quite easily by an individual student. Therefore, the specific problems should be open real-world tasks that include *reference contexts* for elementary students. The wording of the problems should not contain numbers in order to avoid that the children start calculating with the given numbers without analysing the context of the specific situation. According to these criteria the following problems were chosen/developed and used in the study:

- How much paper does your school use in a month? (*paper problem*)
- Your class is planning a trip to visit the Cologne Cathedral. Is it better to travel by bus or by train? (*cathedral problem*)
- How many children are as heavy as a polar bear? (*polar bear problem*)
- There is a 3 km tailback on the A1 motorway between Muenster and Bremen. How many vehicles are caught in this traffic jam? (*traffic problem*)

Discussion of Selected Results

Due to the complexity of the complete study, in this section selected examples will be introduced in order to illustrate the insights that were gained with respect to occurring types of interaction and their classification. A cursory viewing of the video recordings of the different group works suggested a rather unstructured and unsystematic solution process—especially in the two groups with low achievers. In addition, the student teachers who had videotaped these groups initially were disappointed with the observed course of the group work that they had perceived as "chaotic" and "haphazard". At the same time they expressed surprise regarding the fact that most groups not only "somehow" managed to generate sensible solutions to all four problems but also were able to present and explain their solution to their peers during the "strategies conference".

The systematic sequential and special interpretations of the transcripts of the video recordings provided some insight into this observation. A re-occurring phenomenon was a kind of "interweaving of thought processes". Figure 5 illustrates, with respect to the *traffic problem,* that each of the three girls follows her own line of thought, while comments, statements or questions of other group members are (subconsciously?) incorporated in the individual considerations. As a result of the group discussion, Alwina reduces her original estimation of "more than 100 cars"—obviously because she realised that the longer a single car is, the lower is the number of cars that "fit" in a 3 km tailback (inverse proportionality).

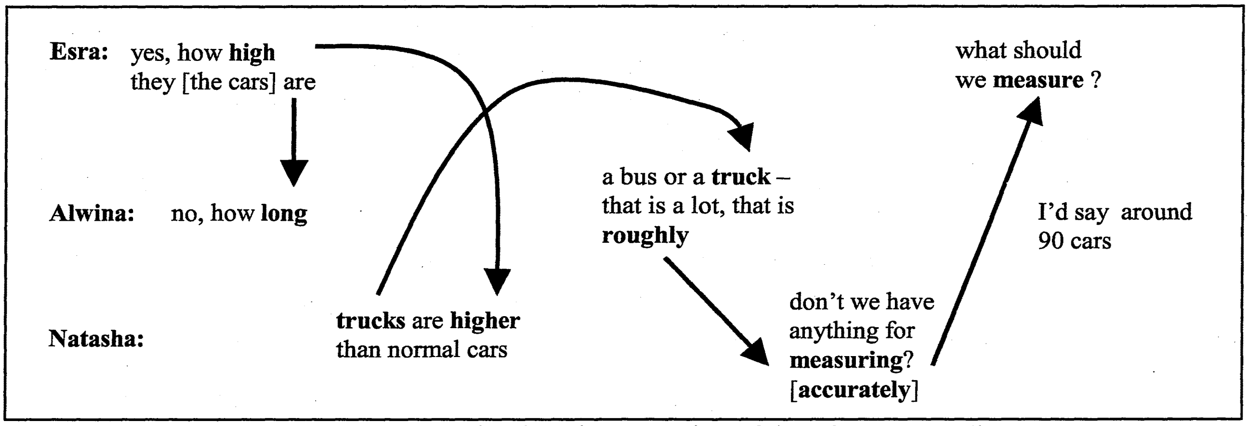

Figure 5. Example of an "interweaving of thought processes" in the interaction three low-achieving fourth graders working on the *traffic problem.*

A major difficulty encountered during the interpretative data analysis however, was the representation of solution processes and group discussions in a way that allowed theoretical generalisations. Transcript excerpts provided little meaningful information in terms of the quality of the interaction process, and the initial attempts of graphical representation as shown above were not satisfactory in terms of distinguishing different types of interaction. The five types of dyadic interaction identified by Kieran and Dreyfus (1998) in this context helped to classify some, however, not all of the phenomena arising from the sequential interpretative analyses. A vast majority of the observed interaction sequences did not match the typical notion of cooperative work in which the group members on the one hand contribute and carefully explain their own ideas and on the other hand try to understand and appropriately react to the suggestions of others. Instead, the group members frequently expressed spontaneous thoughts, guesses and estimations as to what the solution could be.

The extended adaptation of the four classes of dyadic interaction in terms of contingency by Jones and Gerard to the group work episodes helped to classify the children's interaction processes in different group settings. Asymmetrical contingency, for example, was found predominantly in mixed-ability groups where participation was frequently dominated by high achievers (e.g., Vera in Fig. 6) who took a leading role with respect to the development and execution of the solution process.

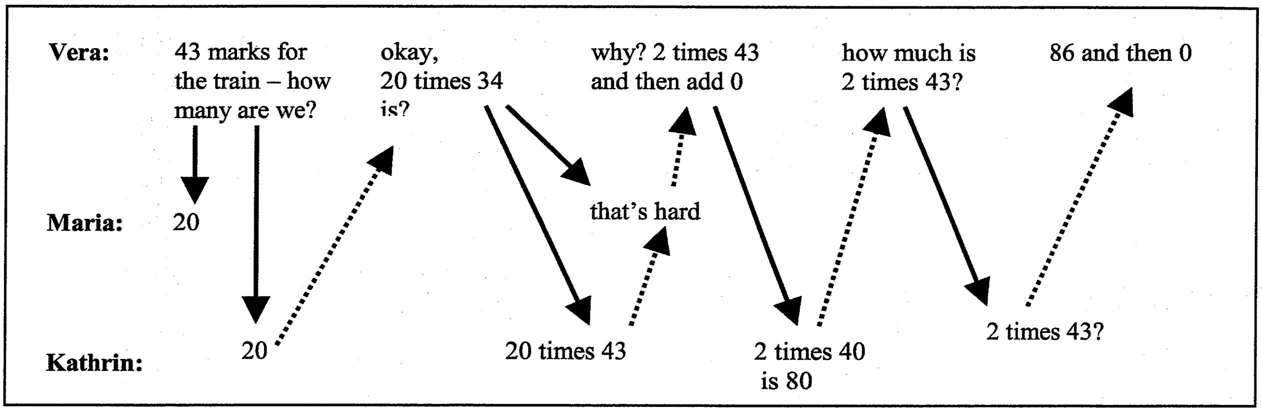

Figure 6. Example of an asymmetrical contingency in a mixed-ability group working on the *cathedral problem.*

Mutual contingency on the contrary, was the class of interaction that was found least frequently and that occurred mainly among high achievers in mixed-ability groups (the data sample did not include a group in which all members were considered high achievers). The majority of interaction sequences that were found throughout the entire data set, in both mixed-ability groups as well as among low achievers, are characterised by reactive contingency, that is, the children reacted (often very spontaneously) to comments from their peers without developing and contributing their own strategies. A typical example of that class of interaction is shown in Figure 7.

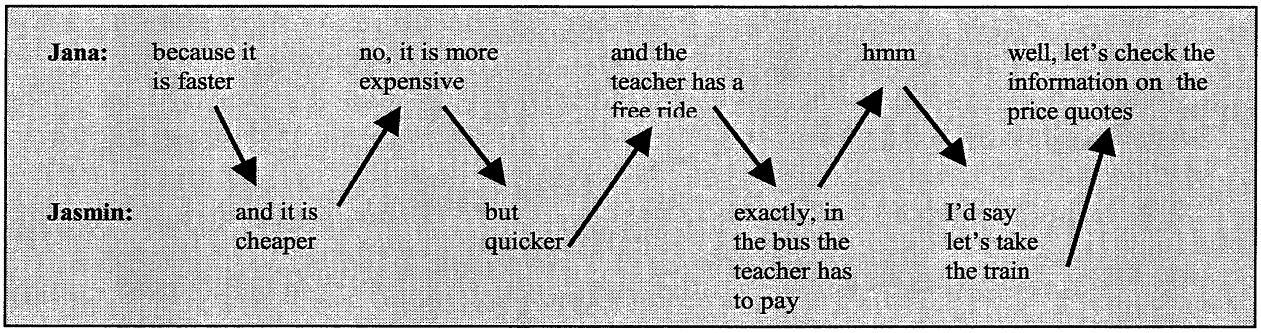

Figure 7. Example of a reactive contingent interaction of two children from a mixed-ability group of four working on the *cathedral problem.*

Conclusions

Since the majority of all group work episodes observed, including the groups of alleged low achievers, proved to be successful (the groups not only found but could also explain a sensible solution), their largely haphazard approaches obviously did not have a negative impact on the quality of the solution. However, group work episodes dominated by reactive interaction were in most cases very time consuming. Furthermore, in these episodes, which often took up to 60 minutes or longer, pseudo-contingent behavior was also a determining factor with respect to group interaction. This might be related to the fact that it is particularly difficult for young children—especially those with mathematical learning difficulties—to enter someone else's "universe of thought" (Trognon, 1993) parallel to their own thinking and mathematical sense making. A further element that was found corresponds to what Kieran and Dreyfus have called "anti-interaction". Supported by verbal signals like "hang on" or "just be quiet" individual members deliberately refused the interaction with others for a period of time in order to be able to follow a new thought and think about it without being distracted or disturbed. Such anti-interactive behavior however, should not necessarily be judged negatively. It indicates that also (or especially?) during group work, phases of individual and undisturbed thinking are crucial for mathematics learning. In order to comprehensively understand the conditions of learning in small group settings in mathematics classrooms, further studies with research designs focussing particularly on the individual learning processes are clearly needed.

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